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Experiments and entropy generation minimization analysis of a cross-flow heat exchanger

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Abstract—This paper presents a cross-flow plate type heat exchanger which has been studied and manufactured in the laboratory conditions because of its effective use in waste heat recovery systems. This new heat exchanger is tested with an applicable experimental set up, considering temperatures, velocity of the air and the pressure losses occurring in the system. These variables have measured and the efficiency of the system has determined. The irreversibility of the heat exchanger has been taken into consideration while the design of the heat exchanger is being performed so that the minimum entropy generation number has analysed with respect to second law of thermodynamics in the cross-flow heat exchanger. The minimum entropy generation number depends on parameters called optimum flow path length, dimensionless mass velocity and dimensionless heat transfer area. Variations of the entropy generation number with these parameters are analysed and introduced with their graphics and comments. © 1997 Elsevier Science Ltd.

INTRODUCTION

The increase in energy cost per unit with energy consumption has required more effective use of energy. In this case, to decrease energy losses and to use lost energy is possibly getting more and more important. The main purpose of waste energy recovery systems is to transfer fresh-air from most waste energy. Heat exchangers are the most important member of waste energy recovery systems. In this study, a recuperative heat exchanger has been examined because it is more applicable in waste energy recovery systems and industrial applications. The recuperative heat exchangers transfer only sensible heat. Heat transfer of recuperative heat exchangers with fixed members occur between a solid surface and the fluids flowing in the same or different directions at different temperatures. The most advantageous point of this type heat exchanger is to have cheaper cost of building, manufacturing, servicing and operating with unmixed fluids. For these reasons, a plate type heat exchanger of the recuperative type was designed, manufactured in laboratory conditions and tested with an applicable experimental set up. Experimental effectiveness of the system has compared with theoretical results and the design criterions were examined based on the second law of thermodynamics. Some of the researches about this subject has been reported below.

Bejan [1], investigated a heat exchanger with two types of losses, as heat transfer losses and frictional pressure drops in channels. Heat transfer losses can be reduced by increasing the heat transfer area, but in this case pressure drops in the channels increase. These two losses determine the irreversibility level of heat exchanger. Bejan reported heat exchanger effectiveness by using the entropy generation number. According to Bejan's study, it is enough to increase the effectiveness by using design criterions such as the minimization of difference wall temperature or maximization of the ratio of heat transfer coefficient to fluid pumping power.

Bejan [2], applied the entropy imbalance equation or entropy generation balance to a control volume of an open system and explained entropy generation, for gas–gas heat exchanger, as the sum of the entropy generation caused by finite temperature difference with frictional pressure drop. Bejan expressed that entropy generation equal to zero corresponds to the highest quality and entropy generation greater than zero represents poorer quality.

Bejan and Poulikakos [3], reported the minimization of entropy generation in forced convection for the design of extended surfaces by the use of the first and second laws of thermodynamics. Entropy generation rate formula was derived for general fins, analytical and graphical results were reported for the minimization of irreversibility.

Eğrican [4], investigated LMTD (logarithmic mean temperature difference) method based on the first law of thermodynamics with effectiveness-transfer unit methods and entropy generation units based on the second law of thermodynamics and applied this method to counter-flow shell and tube heat exchanger, giving an example.

Sekulic [5], examined the quality of energy trans-

NOMENCLATURE

a	half bottom side of plate [m]
A	total heat transfer area [m^2]
A_c	duct cross-sectional area [m^2]
C	heat capacity of fluid [W K^{-1}]
c_p	specific heat capacity [J kg^{-1} K^{-1}]
d	diameter [m]
D	duct hydraulic diameter [m]
f	friction factor
F	heat transfer surface area [m^2]
G	mass velocity [kg s^{-1} m^{-2}]
K	overall heat transfer coefficient [W m^{-2} K^{-1}]
L	length of duct, length of heat exchanger [m]
m	mass flow rate [kg s^{-1}]
Nu	Nusselt number
N_s	entropy generation number
N_{tu}	number of transfer unit
Pr	Prandtl number
Q	heat transfer rate [W]
R	ideal gas constant [J kg^{-1} K^{-1}]
Re	Reynolds number
s	plate thickness [m]
S	entropy (W/K).

Greek symbols

ΔP	pressure drop [Pa]
ΔT	temperature difference [K]
ρ	mean density of fluid in duct [kg m^{-3}]
ε	effectiveness.

Indices

*	dimensionless
1	hot fluid
2	cold fluid
max	maximum
min	minimum
i	inlet
o	outlet.

formation of a heat exchanger based on the second law of thermodynamics and depended this transformation on different parameters such as inlet temperatures ratio, fluid flow heat capacity rate ratio and effectiveness of heat exchanger. The influence of these parameters on the quality of energy transformation was examined for different types of heat exchangers and obtained maximum entropy generation with an equation for cross-flow heat exchanger both fluids unmixed.

Van Den Bulck [6], investigated optimal design of cross-flow heat exchanger and determined optimal distribution of transfer area for maximum effectiveness of heat exchanger.

A Bejan's number was defined based on the duct entropy generation rates due to the end-to-end pressure drop and the wall-stream temperature difference for the second law analysis of a heat exchanger [7]. It was reported that this dimensionless number is essential in at least four areas of heat transfer: electronic cooling, scale analysis of forced convection, second law analysis of heat exchangers, and contact melting and lubrication [7].

PLATE TYPE HEAT EXCHANGER AND EXPERIMENTAL SET UP

Plate layers with fresh and waste air channels were separated as airtight in the plate-type heat exchanger with fixed members. Heat transfers directly between hot waste air and cold fresh air flow. This kind of heat exchangers generally operate cross-flow principle. The heat exchanger, which is manufactured and tested, is

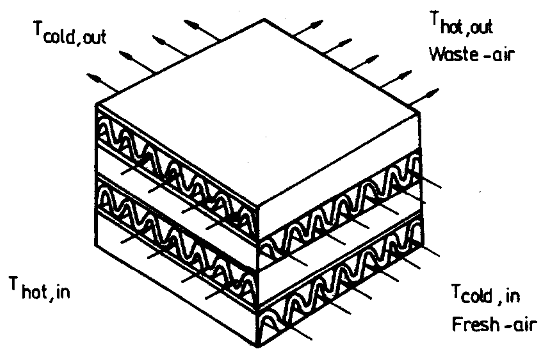

Fig. 1. Cross-flow plate type heat exchanger.

shown in Fig. 1 as an example. As seen in Fig. 1, while the waste air is passing from one direction of the channels, the fresh air is also passing from the other direction of the channels. The waste air transfers its heat to the fresh air during this operation of cross-flow plate type heat exchanger. Because of the air transfers in different channels, the heat transfer area is quite large. The waste air and the fresh air can be used together in this type of exchanger, because these air flows are passing in different channels. In case of using dirty and dusty air, the waste air should be passed through a filter for the prevention of thermal resistance. Channel material and geometry of the plate type heat exchanger can be chosen from different characteristics and forms. The heat exchanger which is designed and manufactured in laboratory conditions has been explained with details in their works [8]. For design of the cross-flow heat exchanger, an

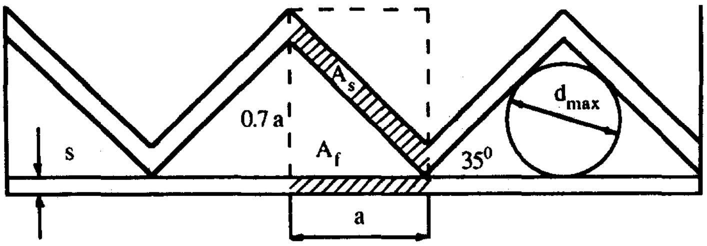

Fig. 2. Channel geometry of heat exchanger.

aluminum plate is used. The thickness of the aluminum plate is 0.35 mm. As seen in Fig. 2, heat transfer mass (matrix) has been considered as an isosceles triangle profile to increase the heat transfer surface and to facilitate manufacturing. The length of bottom plate is an isosceles triangle and its sides were designed as $2a$ and $1.22a$, respectively ($a = 1.7$ mm) [9]. Isosceles triangle profile plates were separated from aluminum flat plates and located overlap in a cube frame. The length of the heat exchanger is 0.35 m.

Figure 3 represents an experimental set up of the manufactured heat exchanger which is established to determine effectiveness of the set up. Inlet and outlet air channels used for the experimental set up was manufactured from galvanized iron plate of 0.7 mm thickness (the dimensions of channels are 40*40 cm). A serpentine was located in the inlet of the channel on the fresh air side of the recuperator. Hot water from the boiler is circulated in the serpentine. In order to obtain linear velocity distribution in the channels and to measure the linear velocity exactly, wire sieves have been placed in front of the serpentine and the outlet of the fan. Throttling clack in experiment set up has performed to measure for different velocities.

Temperature and air velocity of the outlet and the inlet fluids of the recuperator were measured continuously to determine the effectiveness of the heat exchanger. Thermocouples and potentiometer were used to measure temperature of the experimental set up and thermocouple was chosen as Fe–constantan.

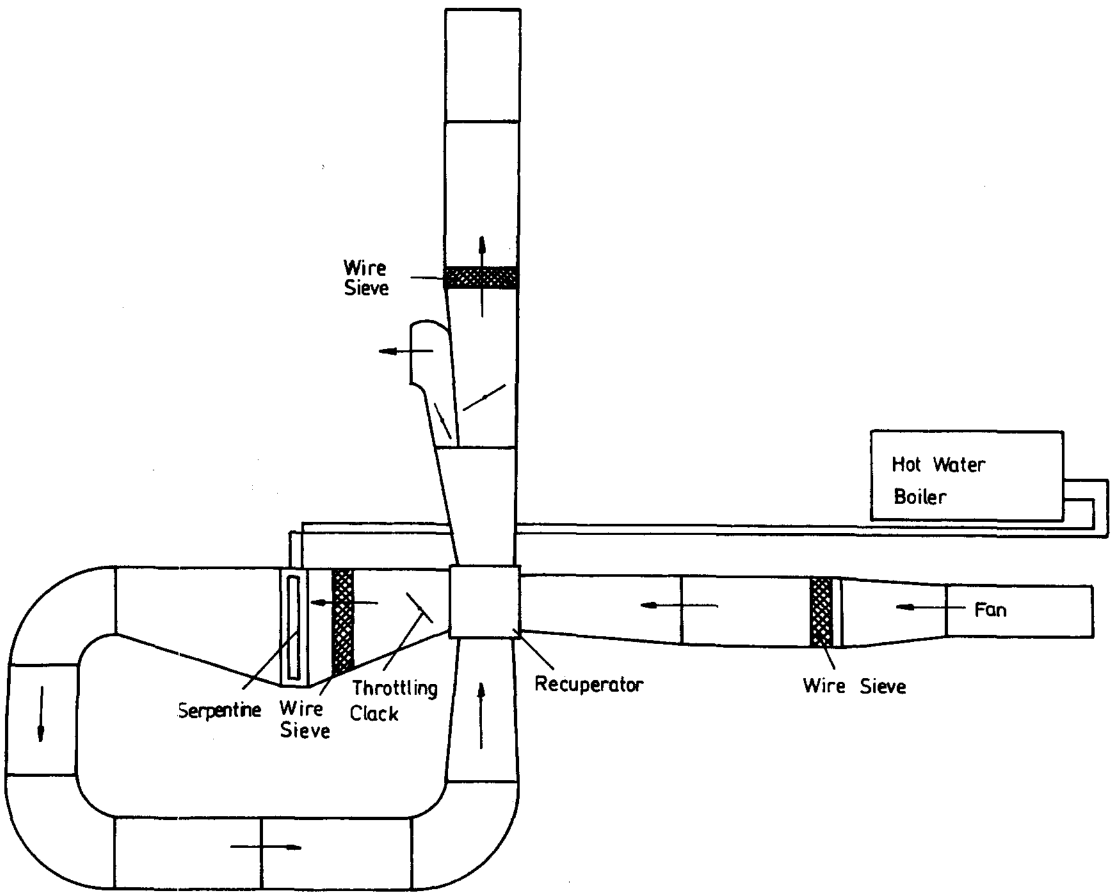

Fig. 3. Experimental set up.

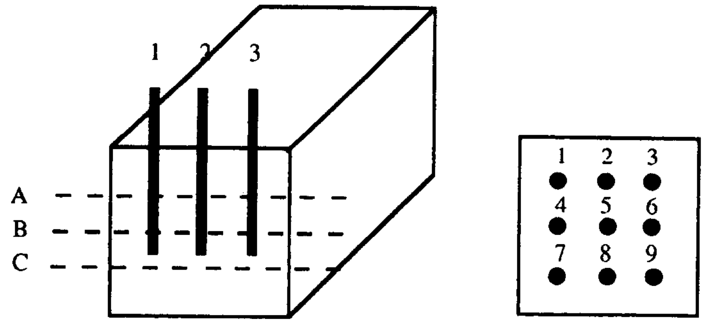

Fig. 4. Measurement points of channels.

Three thermocouples were put in any channel and experimental results were obtained by scanning on three different plane in the same cross-section. Thus measurements have been occurred for nine points in same cross-section and arithmetic means of these nine values were calculated. An analog velocitymeter was used to measure air velocity in the channels. Air velocity is also measured with the same principle used to measure temperature (Fig. 4). Experimental results given in Tables 1 and 2 were determined when clack was open and half-open, respectively. The inlet and the outlet points of the heat exchanger were shown in Fig. 5.

Table 2. Measurement results (clack is half-open and water heat is 60, 70, 80°C)

Measurement results	60°C	70°C	80°C
$T_{1,i}$ (°C)	34.6	33.2	30.9
$T_{2,i}$ (°C)	50.3	56.7	49.0
$T_{1,o}$ (°C)	42.7	45.4	40.8
$T_{2,o}$ (°C)	38.3	39.6	35.0
$V_{1,i}$ (m^3/s)	0.18	0.19	0.19
$P_{1,i}$ (Pa)	1308.4	1304.5	1300.6
$P_{2,i}$ (Pa)	503.2	507.1	499.4
$P_{1,o}$ (Pa)	681.3	677.4	673.6
$P_{2,o}$ (Pa)	7.8	7.8	7.8
Effectiveness (%)	72.9	71.3	70.2

Table 1. Measurement results (clack is open and water heat is 60, 70, 80°C)

Measurement results	60°C	70°C	80°C
$T_{1,i}$ (°C)	33.6	32.2	28.8
$T_{2,i}$ (°C)	50.5	58.3	59.5
$T_{1,o}$ (°C)	42.9	46.5	45.6
$T_{2,o}$ (°C)	37.6	37.7	35.2
$V_{1,i}$ (m^3/s)	0.19	0.2	0.2
$P_{1,i}$ (Pa)	1312.3	1308.4	1304.5
$P_{2,i}$ (Pa)	588.4	580.7	580.7
$P_{1,o}$ (Pa)	592.3	588.4	576.8
$P_{2,o}$ (Pa)	7.8	7.8	7.8
Effectiveness, (%)	69	68.9	68.8

THE SECOND LAW ANALYSIS OF HEAT EXCHANGER

The first law of thermodynamics states that energy can be neither created nor destroyed; it can be only change forms. The first law of thermodynamics is concerned with the quality of energy and the transformations of energy from one form to another with no regard to its quality [10]. According to the second law of thermodynamics, losses always occur in real cycles because of irreversibilities and energy has poorer quality. As a result of this, entropy increases. That is way it is necessary to use an analysis based on second law of thermodynamics in order to use of energy effectively and to minimize entropy generation.

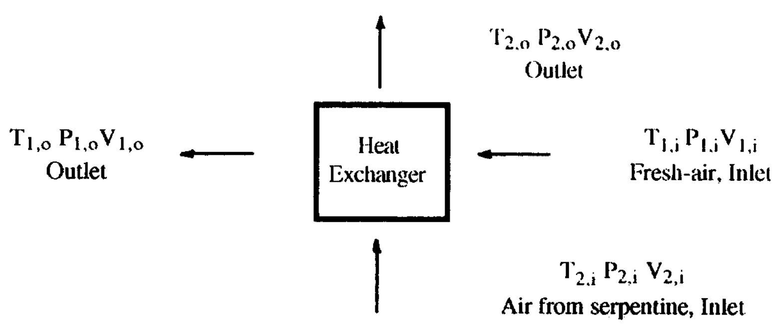

Fig. 5. Outlet and inlet points of heat exchanger.

The outlet temperatures and the total heat transfer from the hot fluid to the cold fluid can be calculated by the use of ε–N_{tu} method, based on the second law of thermodynamics, when the inlet temperatures of the hot and the cold fluid, mass flow rates of the fluids, physical properties of fluids and the type of the heat exchanger are specified. Effectiveness ε is defined as the ratio between actual heat transfer rate and the maximum possible heat transfer rate from one stream to another. Effectiveness ε is given by Bayazıtoğlu and Özışık [11] as

$$\varepsilon = \frac{Q}{Q_{max}}. \tag{1}$$

Here maximum possible heat transfer rate Q_{max} is determined as

$$Q_{max} = (\dot{m}c_p)_{min}(T_{1,i} - T_{2,i}). \tag{2}$$

The symbols 1 and 2 refer to the hot and the cold fluids. The actual heat transfer rate Q is defined as

$$Q = (\dot{m}c_p)_1(T_{1,i} - T_{1,o}) = (\dot{m}c_p)_2(T_{2,o} - T_{2,i}). \tag{3}$$

Where we define $(mc_p)_1 = C_{hot}$, $(mc_p)_2 = C_{cold}$, and C_{min} is smaller of C_{hot} and C_{cold}. Thus effectiveness can be written again from equations (2) and (3)

$$\varepsilon = \frac{C_{hot}(T_{1,i} - T_{1,o})}{C_{min}(T_{1,i} - T_{2,i})} \tag{4}$$

$$\varepsilon = \frac{C_{cold}(T_{2,o} - T_{2,i})}{C_{min}(T_{1,i} - T_{2,i})}. \tag{5}$$

Here C_{min} means the smaller of heat capacity for hot and cold fluids. Effectiveness for $C^* = 1$ can be rewritten as

$$\varepsilon = \frac{(T_{1,i} - T_{1,o})}{(T_{1,i} - T_{2,i})} = \frac{(T_{2,o} - T_{2,i})}{(T_{1,i} - T_{2,i})}. \tag{6}$$

The outlet temperatures of both fluids can be calculated from equation (6) with the use of the effectiveness and the inlet temperatures of both fluids.

$$T_{1,o} = T_{1,i} - \varepsilon(T_{1,i} - T_{2,i}) \tag{7}$$

$$T_{2,o} = T_{2,i} + \varepsilon(T_{1,i} - T_{2,i}). \tag{8}$$

The number of transfer units is denoted by N_{tu} and defined as

$$N_{tu} = \frac{AK}{C_{min}}. \tag{9}$$

The number of transfer units is the ratio between heat capacity of the exchanger and heat capacity of the flow. The actual heat transfer area and the overall heat transfer coefficient is denoted as A and K, respectively. Effectiveness of the cross-flow heat exchangers depends on C^* and N_{tu} parameters given by Bayazıtoğlu and Özışık [11].

$$\varepsilon = 1 - \exp\left\{\frac{1}{C^*}(N_{tu})^{0.22}[\exp(-C^*(N_{tu})^{0.78}) - 1]\right\}. \tag{10}$$

A heat exchanger is characterized by two types of losses, as temperature difference and frictional pressure drop in the channels. These losses refer to irreversibility quantity and some methods are investigated for minimizing of these losses [2]. Bejan, examined a parameter called entropy generation number for minimizing both losses and described this parameter as the ratio between entropy generation rate and the heat capacity rate with equation (11).

$$N_s = \frac{\dot{S}}{C}. \tag{11}$$

The entropy generation number $N_s \to 0$ implies that these losses approach zero and these losses increase when N_s has high values. Entropy generation can be written considering a balance cross-flow ($C^* = 1$) in which temperature difference and frictional pressure drops are not ignored with respect to Fig. 6.

Thus the entropy generation rate is

$$\dot{S}_{gen} = \dot{m}c_p \ln\frac{T_{1,o}}{T_{1,i}} + \dot{m}c_p \ln\frac{T_{2,o}}{T_{2,i}} - \dot{m}R\ln\frac{P_{1,o}}{P_{1,i}} - \dot{m}R\ln\frac{P_{2,o}}{P_{2,i}} \tag{12}$$

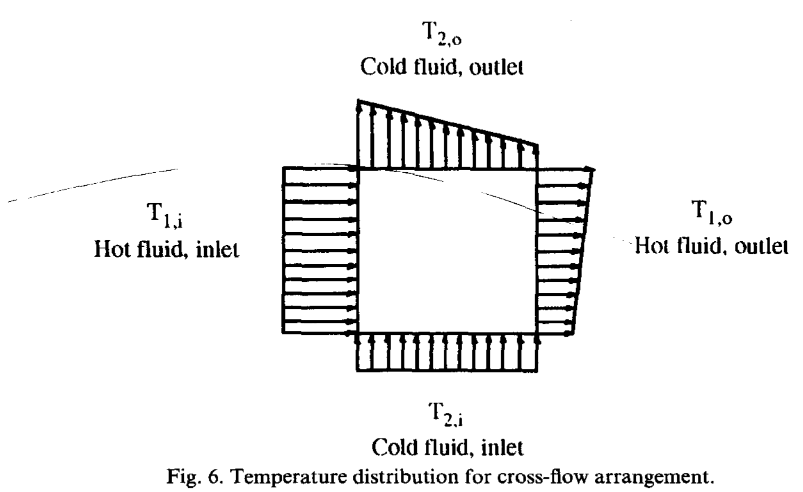

Fig. 6. Temperature distribution for cross-flow arrangement.

where the first two terms on the right-hand side of the equation represent the heat transfer irreversibility and the last two terms represent the fluid friction.

The following equation can be written for outlet pressure.

$$P_{1,o} = P_{1,i} - \Delta P_1 \quad P_{2,o} = P_{2,i} - \Delta P_2. \tag{13}$$

So equation (12) can be rewritten by using equation (13) as

$$N_s = \frac{\dot{S}}{C} = \ln\frac{T_{2,i}}{T_{1,i}}\left[1-(1-\varepsilon)\frac{T_{2,i}-T_{1,i}}{T_{2,i}}\right] + \ln\frac{T_{1,i}}{T_{2,i}}\left[1+(1-\varepsilon)\frac{T_{2,i}-T_{1,i}}{T_{1,i}}\right] - \frac{R}{c_p}\ln\left[1-\left(\frac{\Delta P}{P}\right)_1\right] - \frac{R}{c_p}\ln\left[1-\left(\frac{\Delta P}{P}\right)_2\right]. \tag{14}$$

$(1-\varepsilon)$ and $(\Delta P/P)_{1,2}$ are considerably smaller than unity for nearly ideal heat exchanger [2]. In this case, the entropy generation number can be rearranged as

$$N_s = (1-\varepsilon)\frac{(T_{2,i}-T_{1,i})^2}{T_{1,i}T_{2,i}} + \frac{R}{c_p}\left[\left(\frac{\Delta P}{P}\right)_1 + \left(\frac{\Delta P}{P}\right)_2\right]. \tag{15}$$

In this study, effectiveness in equation (10) is rearranged for balanced cross-flow arrangement and the term $(1-\varepsilon)$ in equation (15) is introduced as

$$1-\varepsilon = 0.477 N_{tu}^{-0.4}. \tag{16}$$

The entropy generation number in equation (15) is converted to the equation below with the definition of dimensionless temperature difference ΔT^*

$$\Delta T^* = \frac{|T_{2,i}-T_{1,i}|}{\sqrt{T_{1,i}T_{2,i}}} \tag{17}$$

$$N_s = 0.477 N_{tu}^{-0.4}(\Delta T^*)^2 + \frac{R}{c_p}\left(\frac{\Delta P}{P}\right)_{1,2}. \tag{18}$$

The entropy generation number should be minimized to decrease heat transfer losses. For that reason the effects of construction dimensions should be considered for any heat exchanger design. Thus entropy generation in equation (18) is rewritten as equation (22), by the use of following equations [2].

$$\frac{\Delta P}{P} = f\frac{4L}{D}\frac{G^2}{2\rho P} \tag{19}$$

$$N_{tu} = \frac{4L}{D}St \tag{20}$$

$$G^* = \frac{G}{\sqrt{2\rho P}}, \quad St = \frac{Nu}{RePr} \tag{21}$$

$$N_s = \frac{0.477(\Delta T^*)^2}{\left(\frac{4L}{D}\right)^{0.4} St^{0.4}} + \frac{R}{c_p}f\left(\frac{4L}{D}\right)G^{*2}. \tag{22}$$

G^*, St and f are described as dimensionless mass velocity, Stanton number and friction factor, respectively, in equation (22).

Optimum flow path length $(4L/D)$ which minimizes the entropy generation number is

$$\left(\frac{4L}{D}\right)_{opt} = \left[0.1908\frac{(\Delta T^*)^2 c_p}{St^{0.4} R f G^{*2}}\right]^{1/1.4}. \tag{23}$$

Thus, equation (22) is reobtained from equation (23):

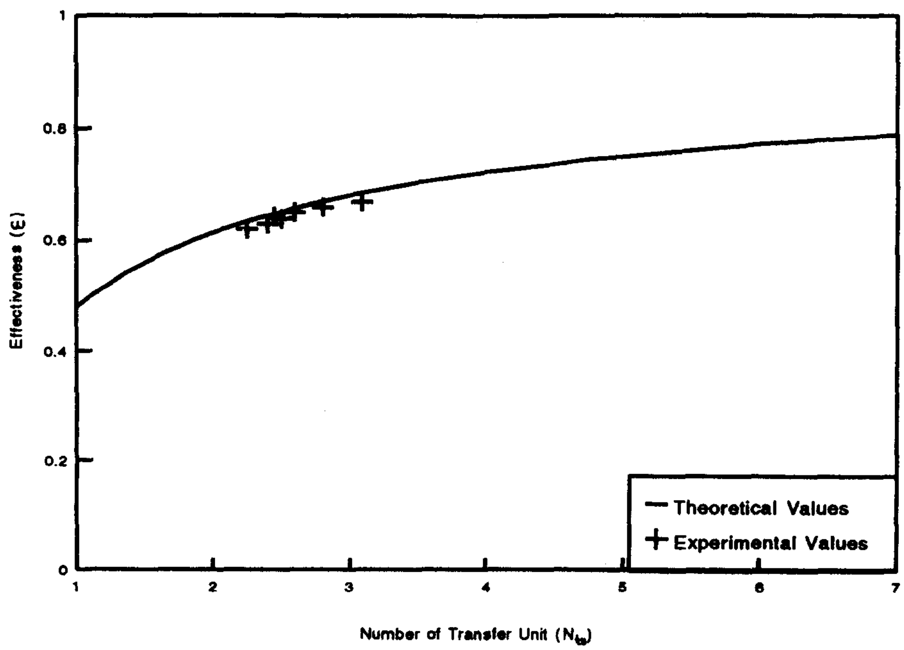

Fig. 7. Variation between effectiveness and transfer unit number.

$$N_{s,\min} = \frac{0.477(\Delta T^*)^2}{\left(\frac{4L}{D}\right)_{opt}^{0.4} St^{0.4}} + \frac{R}{c_p} f \left(\frac{4L}{D}\right)_{opt} G^{*2}. \quad (24)$$

$N_{s,\min}$ represents the minimum entropy generation number. The effects of the heat transfer area A and mass velocity G can be analysed to minimize the entropy generation number. In this case, heat transfer area A and duct cross-sectional area A_c are written from the definition of hydraulic diameter below.

$$A = \frac{4L}{D} A_c. \quad (25)$$

So, dimensionless heat transfer area A^* is defined from equation (25) as

$$A^* = \frac{4L}{D} \frac{1}{G^*}. \quad (26)$$

Entropy generation number given with equation (22) is determined as

$$N_s = \frac{0.477(\Delta T^*)^2}{St^{0.4} A^{*0.4} G^{*0.4}} + \frac{R}{c_p} f A^* G^{*3}. \quad (27)$$

Optimum dimensionless mass velocity G^* which minimizes the entropy generation number can be determined as

$$G^*_{opt} = \left[\frac{0.1908(\Delta T^*)^2}{3A^{*1.4} St^{0.4} \frac{R}{c_p} f}\right]^{1/3.4}. \quad (28)$$

In consequence, minimum entropy generation number is obtained with equation (28) as

$$N_{s,\min} = \frac{0.477(\Delta T^*)^2}{St^{0.4} A^{*0.4} G^{*\,0.4}_{opt}} + \frac{R}{c_p} f A^* G^{*\,3}_{opt}. \quad (29)$$

DISCUSSIONS AND RESULTS

Variations of the effectiveness of the heat exchanger, which is manufactured in the laboratory conditions with N_{tu} parameter has been given in Fig. 7. As seen from Fig. 7, the experimental effectiveness values for balanced cross-flow arrangement ($C^* = 1$) are suitable for theoretical values. The entropy generation number must be minimized for decreasing the heat transfer losses and the frictional pressure drop in order to increase the effectiveness of the heat exchanger. For that reason, the construction dimensions which minimize entropy generation number can be taken into consideration. The variation of optimum flow path length ($(4L/D)_{opt}$) in equation (23) with dimensionless mass velocity (G^*) was determined considering $\Delta T^* = 0.082$. That variation and experimental measurement is introduced in Fig. 8. As seen from Fig. 8, increase in dimensionless mass velocity or increase in dimensionless temperature difference ensure that the smaller optimum flow path length for heat transfer and pressure drop. This means that we have to have a smaller heat exchanger. The variation of the minimum entropy generation number obtained from this optimum flow path length is shown in Fig. 9 with its experimental value. The higher optimum flow path length, in which G^* is small, ensures more suitable minimum entropy generation number because of the less pressure drop in ducts. Increase in $(4L/D)_{opt}$ means increasing of L or D. Consequently, it can be said that increase in $(4L/D)_{opt}$ causes frictional irreversibility and the decrease in $(4L/D)_{opt}$ leads heat transfer irreversibility. The heat transfer area should be taken into consideration in order to ensure the minimum entropy generation, so that the variations of the dimensionless heat transfer area described in equation (26) with $(G^*)_{opt}$ in equation (28) and exper-

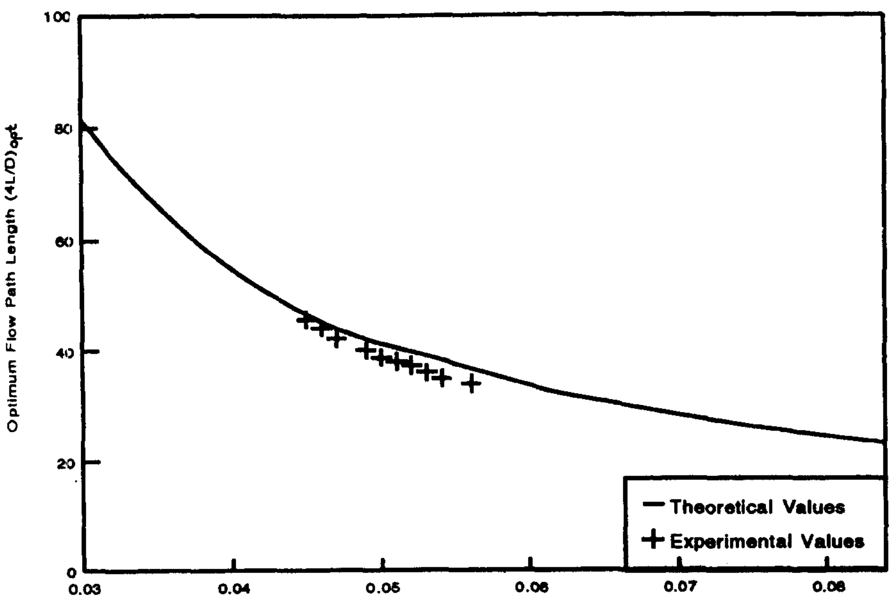

Fig. 8. Variation between optimum flow path length and dimensionless mass velocity.

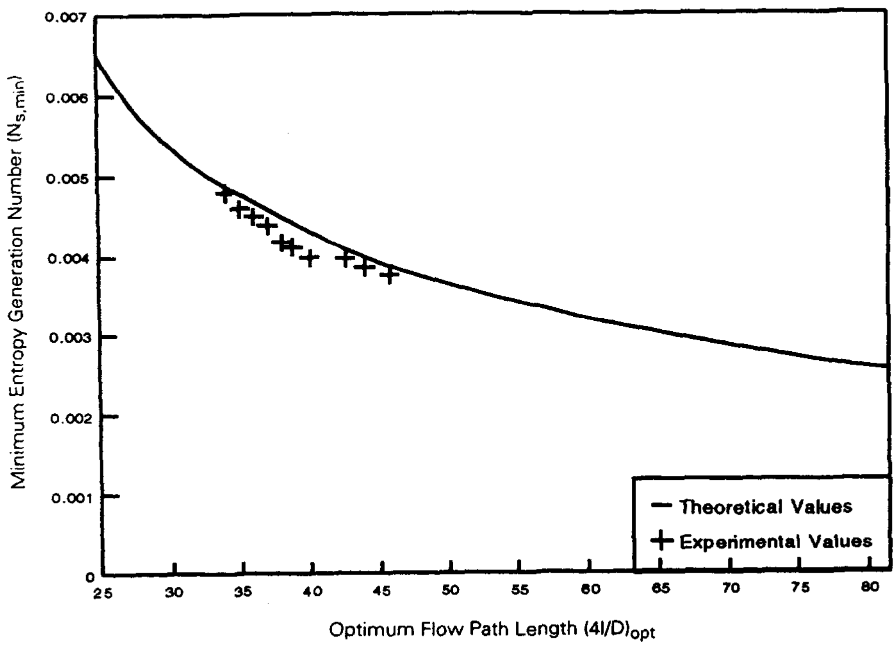

Fig. 9. Variation between minimum entropy generation number and optimum flow path length.

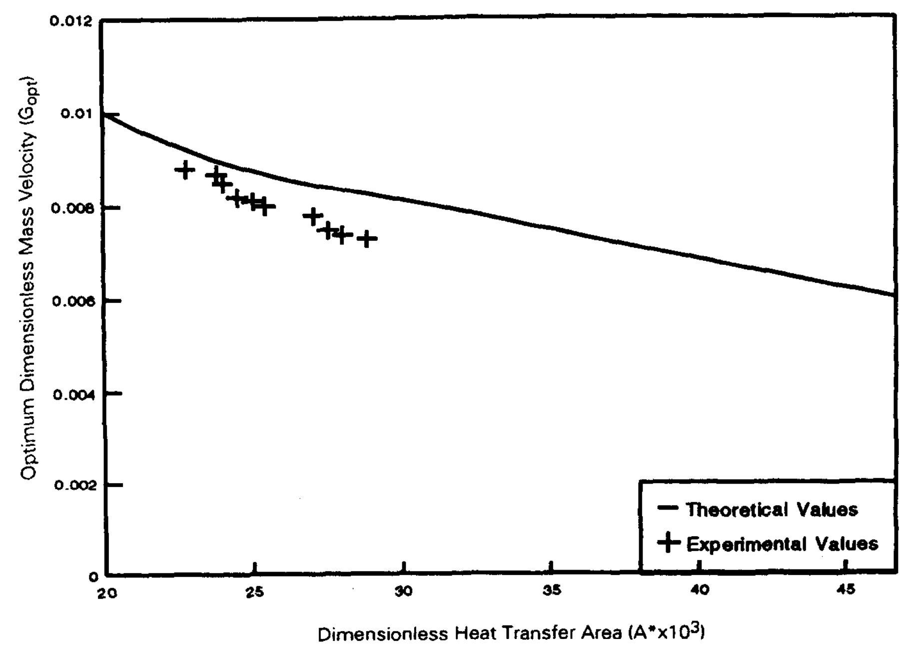

Fig. 10. Variation between optimum dimensionless mass velocity and dimensionless heat transfer area.

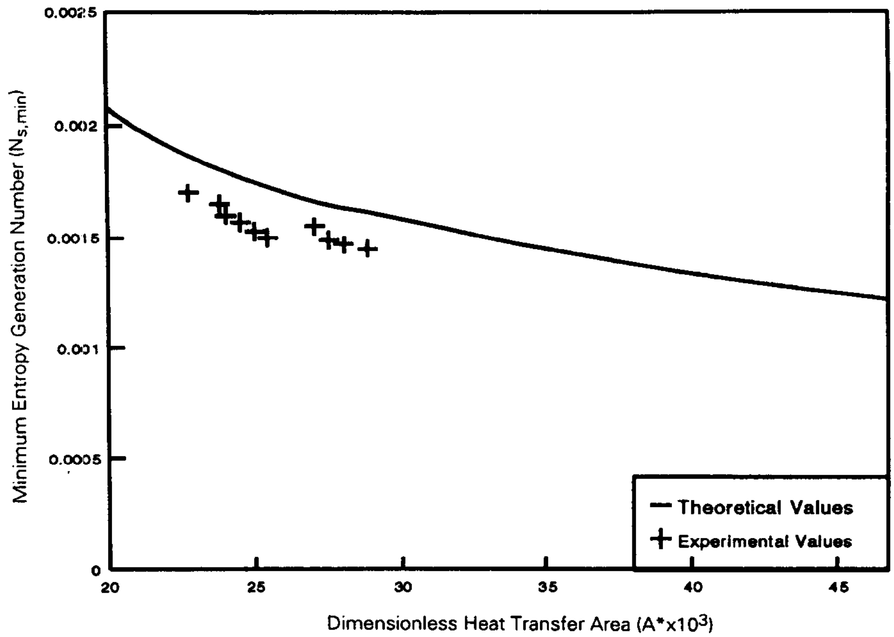

Fig. 11. Variation between minimum entropy generation number and dimensionless heat transfer area.

imental results have been given in Fig. 10. As seen from Fig. 10 $(G^*)_{opt}$ for the heat transfer decreases when dimensionless heat transfer area increases. The effect of dimensionless heat transfer area on the minimum entropy generation number has been given in Fig. 11 with measurement value. It can be seen that the heat transfer area necessity for low mass velocity is more applicable for the minimum entropy generation number. For that reason, the surface area that can decrease the dimensionless heat transfer should be chosen. As seen from all figures, the measurement results are close to the theoretical results. The causes of errors between the theoretical and the experimental values can be considered because of setting and measurements on the system in the laboratory conditions.

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